



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

277. Proposed by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

It is tacitly assumed in elementary geometry that as the number of sides of a regular polygon inscribed in a circle is increased, *in any manner*, that its perimeter has a fixed limit. Beginning with a square and then continually doubling the number of sides we get for the perimeter $2^{n+2}\sqrt{2-E^n(0)}$, where $E(x) \equiv \sqrt{2+x}$. Beginning with a hexagon we get $2^{m+1}3\sqrt{2-E^m(1)}$. The definition of the length of a circle assumes that these expressions have the same limit as $n \doteq \infty$ and $m \doteq \infty$. Prove it.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Putting $\cos x = b$, we have $\sin \frac{1}{2}x = \sqrt{\frac{1}{2}(1-b)} = \frac{1}{2}\sqrt{2-2b}$, $\cos \frac{1}{2}x = \frac{1}{2}\sqrt{2+2b}$, $\sin \frac{1}{4}x = \frac{1}{2}\sqrt{2-\sqrt{2+2b}}$, $\cos \frac{1}{4}x = \frac{1}{2}\sqrt{2+\sqrt{2+2b}}$, $\sin \frac{1}{8}x = \frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2+2b}}}$, etc. Let the radius of a circle $= 1$, then we may represent the perimeter of a regular polygon by $2^{n+2}\sin \frac{1}{2^n}x$. For $n = \infty$, this assumes the indeterminate form $\infty \doteq 0$; but we may reduce it to $\frac{2^2 \sin (1/2^n)x}{1/2^n}$.

Differentiating numerator and denominator with reference to n , we get $2^2 x \cos(1/2^n)x$. For $n = \infty$, this becomes $2^2 x$. For $x = 90^\circ$, $2^2 \times 90 = 360^\circ$.

In regard to the hexagon, we have similarly, $\frac{3 \sin(1/2^n) \times 120^\circ}{1/2^n}$, which after differentiation as above, becomes $3 \times 120^\circ = 360^\circ$, so that the limit in both cases is 360° .

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$E^n(0) = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}};$$

$E^m(1) = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}}$, the last term in the root being $\sqrt{3}$.

$$\text{Let } E^n(0) = E^m(1) = x. \quad \therefore \sqrt{2+x} = x, \text{ or } x=2.$$

$$\frac{2^{n+2}\sqrt{2-E^n(0)}}{2^{m+1}3\sqrt{2-E^m(1)}} = \frac{2^{n+2}\sqrt{2-x}}{2^{m+1}\sqrt{18-9x}} = \frac{2^{n+1}\sqrt{8-4x}}{2^{m+1}\sqrt{18-9x}}.$$

$$\frac{2^{n+1}}{2^{m+1}} \cdot \frac{\sqrt{8-4x}}{\sqrt{18-9x}} = \frac{\sqrt{8-4x}}{\sqrt{18-9x}}, \text{ when } n=m=\infty.$$

$$\text{If } \frac{\sqrt{8-4x}}{\sqrt{18-9x}} = 1, \quad 8-4x = 18-9x, \text{ or } x=2,$$

the same value as found above for x . Hence the expressions are equal when n and m are indefinitely increased.

279. Proposed by C. C. WENTWORTH, C. E., Roanoke, Va.

To construct geometrically the maximum equilateral triangle circumscribed about a given triangle.